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Consequences of Scaling for Neutrino Interactions

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ABSTRACT

It is shown that the scaling of all three structure functions implies bounds for the mean values of E_μ/E_ν and $Q^2/2ME_\nu$. When the bounds are analyzed in view of the recent CERN data, they become so restrictive that they essentially reduce to equalities. Bounds for $Q^2/2ME_\nu$ are translated into bounds for the integral $\int x\, F_2^{\nu N}(x)dx$ and are compared with the corresponding integral in electroproduction. Effects induced by the presence of an intermediate vector boson or heavy leptons are also analyzed.

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I. INTRODUCTION

The objective of this work is to appeal at the scaling phenomenon and extract from the Gargamelle data[1, 2] several quantities relevant to theory and at the same time to obtain bounds for quantities, which are independent of the incident fluxes. The latter is achieved by considering the mean values

$$<f(Q^2, E\mu)> \equiv \frac{\int f(Q^2, E\mu) \frac{d\sigma}{dQ^2 dE\mu} \, dQ^2 dE\mu}{\sigma_{tot}}$$

where $f(Q^2, E\mu)$ can be chosen to be $E\mu / E\nu$, $Q^2/2ME_\nu$ .... It is shown that the scaling hypothesis for either νW_2 or all three structure functions implies upper and lower bounds for such quantities. Violation of the bounds may arise either from a breakdown of scaling or from the presence of a new effect, like violation of locality for the leptonic current or the presence of a W-boson or a heavy lepton.

The virtue of this approach may at first look questionable because detailed information is lost, when we are considering averages. Obviously, precise knowledge of the three structure functions separately would render the present work totally unnecessary. However, such knowledge will be accumulated slowly and a good deal can be learned in the meanwhile, by studying the properties of the mean values discussed here.

The objective of this work is further facilitated by the recent experimental results from the Gargamelle collaboration,[1,2] which suggest that simple relations may exist between the structure functions. In fact, if the ratio of the total cross section $\sigma^{\bar{\nu}}/\sigma^{\nu}$ on isoscalar targets lies in the neighborhood of the lowest bound allowed by scaling, then the bounds for several quantities become so stringent, that for all practical purposes they are equalities. Examples of the results are the following: (α) in neutrino experiments the muon carries on the average half of the incident energy; (β) in antineutrino experiment the muon carries on the average three-quarters of the incident energy; (γ) in neutrino experiments the mean value of Q^2 is approximately 1/9 of $S = 2ME_{\nu}$.

An outline of the paper now follows. In Section II we derive bounds for $\langle E\mu/E\nu\rangle$ assuming that either one or all three of the structure functions scale. A brief discussion of earlier calculations is also presented.[3,4] In section III bounds for the mean value of the square of the momentum transfer[5] are obtained. In Section IV we analyze the bounds in view of the recent CERN data.[1,2] It is established that several of the bounds become so stringent that they are essentially equalities. Deviations from the mean values are estimated in terms of the deviation of the ratio $\sigma^{\bar{\nu}}/\sigma^{\nu}$ from 1/3. The available data also make possible an estimate of the integral $\int F_2^{\nu}(x)\, x dx$. A comparison with the corresponding integral in electroproduction indicates good agreement with the prediction of the "parton-model". In the last section we discuss the effects on the mean values induced by the presence of either a W-boson or a heavy lepton.

II. BOUNDS FOR THE MEAN MUON ENERGY

One of the first quantities to be measured in the reactions

$$\nu + p \rightarrow \mu^- + \text{anything}$$

$$\bar{\nu} + p \rightarrow \mu^+ + \text{anything}$$

is the mean energy of the muon

$$\langle E\mu / E\nu \rangle \equiv \frac{\int \left(\frac{E\mu}{E\nu}\right) \frac{d\sigma}{dQ^2 d\nu}\, dQ^2 d\nu}{\sigma_{tot}} . \qquad (1)$$

Such a ratio requires measurement of the energy of the muon and final hadrons. This may not be trivial, but it is considerably easier than the separation of the structure functions, because it does not require any knowledge of the incident flux and it also integrates over large regions of phase-space. Consequently, accurate measurements can be made even with limited statistics.

We show in this section that Bjorken's scaling hypothesis for all three functions implies

$$\frac{1}{2} \leq \langle E\mu / E_\nu \rangle \leq \frac{3}{4}. \qquad (2)$$

This result may be obtained readily by following the same line of reasoning as that used in Ref. 6, in order to bound $\sigma^{\bar{\nu}}/\sigma^{\nu}$. Assuming the scaling of all three structure functions, the total cross section can be represented in the form:

$$\sigma^\nu = \frac{G^2 ME}{\pi} \int_0^1 dy \int_0^1 dx\, F_2(x) \{(1-y) + y(L) - y(1-y)\,(R)\} \qquad (3)$$

where in the standard notation

E is the neutrino energy

E' is the muon energy

$\nu = E - E'$

$-Q^2$ is the square of the four momentum transfer

$y = \nu / E, \quad x = Q^2 / 2M\nu$

$$(L) = \frac{\sigma_L}{\sigma_L + \sigma_R + 2\sigma_s} \leq 1, \quad (R) = \frac{\sigma_R}{\sigma_L + \sigma_R + 2\sigma_s} \leq 1 \qquad (4)$$

with σ_L, σ_R, $2\sigma_s$ being the cross sections for the absorption of the left-handed, right-handed and scalar current, respectively. It also follows that the quantities (L) and (R) depend only on the variable x.

The same argument can be repeated for the mean muon energy, where after integration over x and y one obtains

$$\left\langle \frac{E'}{E} \right\rangle = \frac{\int \frac{E'}{E} \frac{d\sigma}{dQ^2 d\nu} \, dQ^2 d\nu}{\sigma_{tot}} = \frac{\frac{1}{3} + \frac{1}{6} \langle L \rangle - \frac{1}{12} \langle R \rangle}{\frac{1}{2} + \frac{1}{2} \langle L \rangle - \frac{1}{6} \langle R \rangle} \qquad (5)$$

where the mean quantities on the right hand side are defined as

$$\langle L, R \rangle = \frac{\int F_2(x) \, (L, R) \, dx}{\int F_2(x) \, dx} \, . \qquad (6)$$

The mean values of the cross section ratios are bounded by

$$0 \leq \langle L \rangle \leq 1, \; 0 \leq \langle R \rangle \leq 1, \; 0 \leq \langle L \rangle + \langle R \rangle \leq 1. \qquad (7)$$

As a result we find the limits of Eq. (2), where the upper bound corresponds to $\langle R \rangle = 1$, $\langle L \rangle = 0$ and the lower bound to $\langle R \rangle = 0$, $\langle L \rangle = 1$.

These considerations may be generalized a little bit, if we assume that only $F_2(x) = \nu W_2$ scales and allow for the possibility that (L) and (R) do not scale. Then

$$\langle \frac{E'}{E} \rangle = \frac{\frac{1}{3} + \frac{1}{6}\langle \tilde{L} \rangle - \frac{1}{12}\langle \tilde{R} \rangle}{\frac{1}{2} + \frac{1}{2}\langle L \rangle - \frac{1}{6}\langle R \rangle} \tag{8}$$

where $\langle \tilde{L} \rangle$ and $\langle \tilde{R} \rangle$ are again less than unity but independent of $\langle L \rangle$ and $\langle R \rangle$. The corresponding bounds now are

$$\frac{1}{4} \leq \langle \frac{E'}{E} \rangle \leq 1 \, . \tag{9}$$

The previous result can also be restated as the ratio of the mean muon energies for neutrino and antineutrino induced reactions. For a target which consists of equal numbers of protons and neutrons, charge symmetry implies $F_2^{\nu}(x) = F_2^{\bar{\nu}}(x)$ and consequently

$$\frac{2}{3} \leq \langle E'/E \rangle_{\nu} \Big/ \langle E'/E \rangle_{\bar{\nu}} \leq \frac{3}{2} \, . \tag{10}$$

It is worth emphasizing the close analogy of the bounds obtained above with the bounds obtained for the ratio of the total cross sections. If all structure functions scale, then the total cross section rises linearly with energy and the ratio of cross sections is a constant. If only νW_2 scales then the ratio of cross sections is again bounded but it is not required to be a constant; in addition the bounds on the mean muon energy are weaker.

In concluding this section we discuss two results for the mean muon energy obtained earlier. Volkov and Folomeshkin[3] calculated $\langle E'/E \rangle$ assuming scaling and particular relations between the structure functions, which are not those suggested by the Gargamelle data. A result close to $<E'/E> \approx \frac{1}{2}$ was obtained. Bjorken[4] derived an expression for a muon inelasticity relevant to cosmic ray experiments. It appears that the mean muon energy in his work is defined differently than the $<E'/E>$ discussed in this paper and the bounds obtained depend explicitly on the initial neutrino spectrum, so that any coincidence is just incidental. The form of the spectrum considered by Bjorken is relevant for experiments with cosmic ray neutrinos but not for accelerator experiments. Finally, if we would like to derive bounds for $<E'/E>$ of Eq. (1) using Bjorken's argument, we find that in addition to locality and the specific form of the spectrum we need the assumption that $<E'/E>$ is independent of the energy of the neutrino.

III. BOUNDS ON THE SQUARE OF THE MOMENTUM TRANSFER AND OTHER QUANTITIES

As was mentioned first by Myatt and Perkins[5] Q^2 should be proportional to 2ME in the scaling region. Existing experimental data at present energies do not contradict this prediction. Here we would like to obtain some limits on the coefficient of proportionality.

For the average value of Q^2 one easily obtains

$$<Q^2/2ME>_\nu = \frac{\frac{1}{6}<x> + \frac{1}{3}<L>^* - \frac{1}{12}<R>^*}{\frac{1}{2} + \frac{1}{2}<L> - \frac{1}{6}<R>} = <\frac{2E'}{M}\sin^2\frac{\theta}{2}> \tag{11}$$

where

$$\langle x\rangle = \frac{\int F_2(x)x\,dx}{F_2(x)dx} \text{ and } \langle L\rangle^* = \langle xL\rangle = \frac{\int F_2(x)x(L)dx}{\int F_2(x)dx} \tag{12}$$

with a similar definition for $\langle R\rangle^*$. We want to maximize and minimize the ratio subject to the conditions

$$0 \le (\langle L\rangle^* \text{ and } \langle R\rangle^*) \le \langle x\rangle \le 1;\ 0 \le \langle L\rangle^* \le \langle L\rangle \text{ and } 0 \le \langle R\rangle^* \le \langle R\rangle. \tag{13}$$

The minimum is attained at $\langle x\rangle = 0$. The maximum for $\langle x\rangle = \langle L\rangle = \langle R\rangle = \langle L\rangle^* = 1$ and $\langle R\rangle^* = 0$ giving

$$0 \le \langle Q^2/2ME\rangle_{\nu,\bar{\nu}} \le 3/5. \tag{14}$$

The antineutrino ratio is obtained by the interchange $R \leftrightarrow L$ and a similar argument. Equations (2) and (14) can be generalized to the case of any positive power of E' and Q^2. One easily finds

$$\frac{1}{n+1} \le \langle (E'/E)^n\rangle_\nu \le \frac{1}{1+\frac{n}{3}} \tag{15}$$

$$\frac{2}{3}\,\frac{1}{(n+2)(n+3)} \le \langle (Q^2/2ME)^n\rangle_\nu \Big/ \langle (Q^2/2ME)^n\rangle_{\bar{\nu}} \le \frac{3}{2}(n+2)(n+3) \tag{16}$$

where Eq. (16) refers to isoscalar targets only.

If one tries to average over negative powers of E' or Q^2 one arrives at divergent integrals. The divergence arises from small values of E'. Such integrals are not of interest, as far as scaling is concerned, because there is no reason to believe in scaling for small values of E'. The different powers of $Q^2/2ME$ and of x measure higher moments of $\int F_i(x)\, x^n dx$ and consequently emphasize different regions of x. Linear combinations of the moments are related to sum rules[7] and perhaps provide an easier determination of them, since nature automatically performs the integrations. The determination of such an integral and its comparison with electroproduction is discussed in Section IV.

IV. BOUNDS AND THE LATEST CERN DATA

Existing experimental data[1, 2] obtained at CERN do not contradict the hypothesis that scaling is already observed. As was mentioned by many authors this is a rather surprising fact since the values of ν, Q^2 are not in fact large. Thus, apparent confirmation of scaling could be just an effect of small statistics. Bounds obtained in the preceeding sections can be checked using all the statistics available. For example, if one assumes that the total cross section[8] rises linearly starting from 2 GeV the ratio of $\sigma^{\bar{\nu}}/\sigma^{\nu}$ is known with rather good accuracy

$$\left(\frac{\sigma\bar{\nu}}{\sigma\,\nu}\right)_{\text{exp}} = 0.377 \pm 0.023 \tag{17}$$

It is close to the lowest bound allowed by scaling.

On the other hand, the statistics are too poor to provide independent convincing evidence in favor of linearly growing cross sections. Moreover, one can argue that at present energies, scaling bounds are in fact violated. Indeed, there is no reason to expect that the elastic cross section and the cross section for isobar production should be included to $\sigma_{\nu,\bar{\nu}}$ when predictions of scaling are checked. However, these particular channels seem to constitute a large fraction of the total cross section of the antineutrino interaction and if they are subtracted from σ_{tot} the ratio of the remaining cross sections seems to violate the bounds.[9] We are unable to prove, however, that the cross section of isobar production should be subtracted. The only lesson, therefore, is the indication that it is very desirable to perform further tests of the scaling hypothesis using the CERN data. Since these data are rather preliminary we do not use them to make numerical calculations but restrict ourselves to listing some predictions.

In view of Eq. (17), it seems reasonable to consider bounds in the case when the ratio of the cross sections is close to 1/3:

$$\frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} = \frac{1}{3}(1+\epsilon)\ , \quad \epsilon \ll 1. \tag{18}$$

This provides the constraint equation

$$\psi + \frac{3}{4}\xi = \frac{3}{8}\epsilon + 0(\epsilon^2) \tag{19}$$

where $\psi = \frac{\langle R \rangle}{\langle L \rangle}$ and $\xi = \frac{\langle S \rangle}{\langle L \rangle}$. Consequently

$$\frac{\langle S \rangle}{\langle L \rangle} \leq \frac{1}{2}\epsilon \approx .06 \tag{20}$$

in agreement with the corresponding ratio in electroproduction and with the prediction of the parton model. Using Eq. (19) we also arrive at the following inequalities:

$$\frac{1}{2} \leq <E'/E>_{\nu} \leq \frac{1}{2} + \frac{1}{12}\,\epsilon \qquad (21)$$

$$\frac{3}{4} - \frac{9}{32}\epsilon \leq <E'/E>_{\bar{\nu}} \leq \frac{3}{4} \qquad (22)$$

$$\frac{2}{3} \leq <E'/E>_{\nu} \Big/ <E'/E>_{\bar{\nu}} \leq \frac{2}{3} + \frac{13}{36}\,\epsilon \qquad (23)$$

$$2\left(1 + \epsilon - \frac{35}{16}\frac{\epsilon}{<x>}\right) \leq <Q^2/2ME>_{\nu} \Big/ <Q^2/2ME>_{\bar{\nu}} \leq 2(1+\epsilon) \qquad (24)$$

provided that $\epsilon \ll <x>$.

Equations (21) and (22) hold for individual proton or neutron targets. For Eqs. (23) and (24) averaging over protons and neutrons is required. These equations seem to be rather restrictive considering the small value of $\epsilon (\sim .132)$. Relation (23) seems particularly useful, since it can be tested by arranging for identical production of neutrinos and antineutrinos and measuring only the energies of μ^+ and μ^-'s. In case the agreement of Eq. (18) with the bounds for the ratio $\sigma^{\bar{\nu}}/\sigma^{\nu}$ is just fortuitous due to the inclusion of isobar production, as discussed at the beginning of this section, then one would expect violations of bounds (21) - (24).

Furthermore, a precise determination of ψ and ξ in the constraint equation (19) provides a test of the parton relation $W_2(V) = W_2(A)$, where V and A indicate the contributions arising from the vector and axial currents, respectively. It is shown in the appendix that the experimental upper limit $<R>/<L> \leq \delta \ll 1$ implies

$$\frac{\int F_2(V)dx}{\int F_2(A)dx} = 1 \pm 4\delta^{1/2} + 0(\delta). \tag{25}$$

This together with Eq. (17) and the Conserved Vector Current hypothesis determines the isovector contribution to electroproduction and consequently the isoscalar term. Present data are consistent with a small isoscalar contribution, but the bounds are not very restrictive. What is still needed are more accurate electroproduction and neutrino data, as well as a determination of the ratio $\xi = \frac{<S>}{<L>}$.

Equation (11) is also useful in determining the integral $\int_0^1 xF_2(x)dx$. To this end we rewrite Eq. (11) in the form

$$\frac{1}{2}<x> = <Q^2/2ME>_\nu \left(1 - \frac{4}{6}<R> - <S>\right) + \left(\frac{2}{3}<S>^* + \frac{5}{12}<R>^*\right) \tag{26}$$

by eliminating $<L>$ and $<L>^*$ with the help of the identities

$$<L> + <R> + 2<S> = 1 \text{ and } <L>^* + <R>^* + 2<S>^* = <x>. \tag{27}$$

Using the constraints of Eqs. (13) and (19) we arrive at

$$<Q^2/2ME>_\nu + \frac{1}{3}\epsilon \geq \frac{1}{2}<x> \geq <Q^2/2ME>_\nu \left(1 - \frac{\epsilon}{2}\right) \tag{28}$$

There is already an experimental[5] evaluation of

$$<Q^2/2ME>_\nu \approx \frac{1}{9}. \tag{29}$$

Thus

$$.14 \gtrsim \int x\, F_2(x)dx \gtrsim .10 \tag{30}$$

where we used the recent CERN[2] result

$$\int F_2(x)dx = .47 \pm .07 \tag{31}$$

In the absence of an explicit form for $F_2(x)$ the moments $\int x^n\, F_2(x)dx$ are valuable, because they provide more detailed tests for several hypotheses. In particular, it is common folklore that $F_2(x)$ for neutrinos can be obtained from the corresponding structure functions for electro-production by virtue of the following hypotheses:

(1) the parton (light-cone) prediction $W_2(V) = W_2(A)$

(2) the parton suggestion[10] that the isoscalar contribution to $F_2^{\gamma p} + F_2^{\gamma n}$ is less than 10%.

A recent test of the hypotheses considered the integral $\int F_2(x)dx$ and established[3] a remarkable agreement between electroproduction and neutrino data. We provide here an additional test, which involves the first moment of $F_2(x)$ evaluated in Eq. (16). The corresponding integral in electroproduction is evaluated using the MIT-SLAC[11] data

$$\int x\, F_2^{\gamma N}(x)\, dx \approx .13\ . \tag{32}$$

The agreement is again good, inspite of the uncertainities associated with with the second significant figure in Eqs. (30) and (32).

V. W-BOSON EFFECTS

There are two effects which will modify the bounds discussed: the presence of either a W-boson or of a heavy lepton. In the case of a W-boson we make the substitution

$$G^2 \rightarrow \frac{G^2}{\left(1 + \frac{S}{M^2 W} xy\right)^2}, \; S \approx 2ME \tag{33}$$

and proceed as before. The mean muon energy now is

$$\left\langle\frac{E'}{E}\right\rangle = \frac{\int dx \, F_2(x) \frac{M_W^2}{Sx} - \int dx \, F_2(x) \left(\frac{M_W^2}{Sx}\right)^2 \ln\left(1 + \frac{Sx}{M_W^2}\right)}{\int dx \, F_2(x) \left(\frac{1}{1 + \frac{Sx}{M_W^2}}\right)}, \tag{34}$$

where again we have set $\langle R\rangle = 0$, $\langle L\rangle = 1$. Figure 1 shows $\left\langle\frac{E'}{E}\right\rangle$ as a function S/M_W^2. In calculating the integrals we again appeal to the electroproduction data and apply the hypotheses on page 13, in addition to setting $\sigma_R = \sigma_S = 0$. We notice in the figure that the deviations from a straight line become quite noticeable at $S/M_W^2 \sim 1 - 2$. At $S/M_W^2 \approx 1.5$ the deviation from $1/2$ is $\sim 30\%$. Such a test is sensitive to a W-boson mass $M_W \sim 1.15\sqrt{ME_{Lab}}$.

The presence of a heavy lepton, on the other hand, produces just the opposite effect. The process now proceeds through the steps

$$\begin{array}{l} \nu + p \rightarrow \ell^* + X \\ \qquad\quad \hookrightarrow \ell + Y. \end{array} \tag{35}$$

In this reaction ℓ^* carries, in the mean, half of the neutrino energy. In the subsequent decay however ℓ carries only a fraction of ℓ^*'s energy. For a decay which is isotropic in the ℓ^* rest frame $\frac{E_{\ell^*}}{E_\ell} = 1/2$ and consequently $<E_\ell / E_\nu>$ can be as small as 1/4.

In case that the intermediate boson is produced at high energies with a sizable leptonic decay mode, it could be detected by the presence of two leptons in the final state. On the other hand the leptonic decays may be greatly suppressed and the signature is now the same as in deep inelastic scattering. In the latter case most of the energy is transfered to the hadrons[12] and the mean energy of the muon is very, very small. Thus each one of the three effects will cause noticeable deviations from the value of $\frac{1}{2}$.

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FIGURE CAPTION

The mean energy of the muon in the presence of an intermediate vector boson as a function of s/M_W^2.

APPENDIX

It is shown here that the bound $\frac{<R>}{<L>} \leq \delta$ provides an estimate of the contribution of the vector current alone and the axial current alone to the integrals in Eq. (25). Such a relation provides a comparison with the parton (light cone) prediction. We first show how the bound $\frac{\sigma_R}{\sigma_L} \leq \beta$ can be translated into a relation between the vector and the axial contributions to the cross section. Then we generalize the result to the appropriate integrals. Let v be the vector contribution alone, a the axial contribution alone and i the V-A interference contribution to σ_R. Then

$$\sigma_{R,L} = v + a \pm i \tag{A-1}$$

and

$$\sigma_R - \beta\sigma_L = (1-\beta)(v+a) + (1+\beta)\, i. \tag{A-2}$$

Using Schwartz's inequality

$$4av \geq i^2 \tag{A-3}$$

we obtain

$$0 \geq \sigma_R - \beta\sigma_L \geq (1-\beta)(v+a) - 2(1+\beta)(av)^{\frac{1}{2}} \tag{A-4}$$

which leads to

$$1 + 2\sqrt{\beta} + 0(\beta) \geq \left(\frac{v}{a}\right)^{\frac{1}{2}} \geq 1 - 2\sqrt{\beta} + 0(\beta) \tag{A-5}$$

We can generalize the result to an integral form. For any positive function g, we can rewrite (A-4) as

$$0 \geq \int g(\sigma_R - \beta\sigma_L)dx \geq (1-\beta)\int g(v+a)dx - 2(1+\beta)\int g(av)^{\frac{1}{2}}dx$$
$$\geq (1-\beta)\int g(v+a)dx - 2(1+\beta)\left[\int gv\,dx\right]^{\frac{1}{2}}\left[\int ga\,dx'\right]^{\frac{1}{2}} \qquad \text{(A-6)}$$

where for the last step we appeal again to Schwartz's inequality in its integral form. The solution of (A-6) provides bounds similar to those in (A-5)

$$\frac{\int gvdx}{\int gadx} = 1 \pm 4\beta^{\frac{1}{2}} + 0(\beta) \qquad \text{(A-7)}$$

In applying these results to the present case we recall that

$$F_2(x) = \frac{1}{2\pi}\,(1-x)Q^2\,(2\sigma_S + \sigma_R + \sigma_L) \qquad \text{(A-8)}$$

so that

$$\delta \langle L \rangle \geq \langle R \rangle \qquad \text{(A-9)}$$

is equivalent to

$$\delta\int(1-x)Q^2\,\sigma_L\,dx \geq \int(1-x)Q^2\,\sigma_R\,dx \qquad \text{(A-10)}$$

By combining (A-6), (A-7) and (A-10) one obtains Eq. (25).

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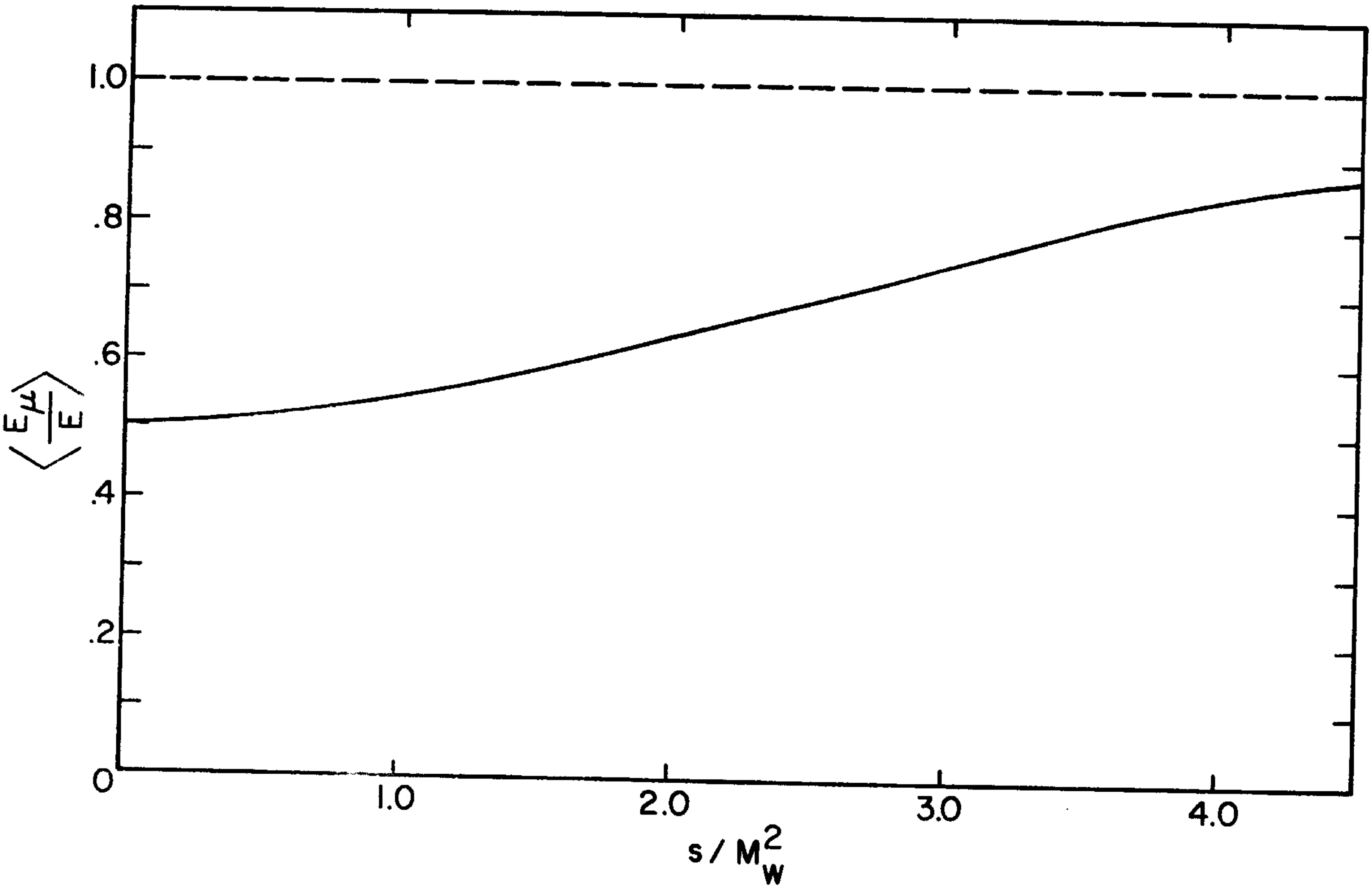

$\left\langle \frac{E_\mu}{E} \right\rangle$
1.0
.8
.6
.4
.2
0
1.0
2.0
3.0
4.0
s / M_W^2